User-centric network fairness through connection-level control

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Abstract—Methods for network resource allocation have mainly focused on establishing fairness among the rates of individual flows. However, since multiple TCP connections in one or many paths can serve a common user, we advocate in this paper a user-centric notion of fairness, which we formulate in the Network Utility Maximization (NUM) framework.

In particular, we develop control laws for the number of connections identified with a certain user, which can include single-path, multipath or more general aggregates of flows, and prove convergence to the optimal resource allocation. This theory applies directly to the case of cooperative users. In the case where connections are generated exogenously by possibly non-cooperative users, we develop admission control policies that ensure both network stability and user-centric fairness.

I. INTRODUCTION

Since the seminal work of Kelly et. al. [1], resource allocation at the transport layer has been modeled as an economic problem, where flow rates are adapted to maximize a network utility objective. This Network Utility Maximization (NUM) framework has also been successfully applied to resource allocation studies across multiple protocol layers (see e.g. [2]).

The economic models are, however, metaphorical: for instance, the “demand curve” that models the response of a TCP flow to congestion “prices” is imposed by the protocol dynamics, not user preferences. Consequently, the ensuing notion of “flow rate fairness” is artificial. Furthermore, it ignores the fact that applications can open multiple connections to obtain a higher rate (see [3] for a sharp discussion of this topic). It becomes apparent that for fairness analysis, a higher level of granularity is required beyond individual TCP connections. As a step in this direction we propose in this paper to allocate resources in terms of the aggregate rate of connections that serve a common “user”, and to explicitly control the number of such connections to achieve “user-centric” fairness.

In previous works [4], [5] we considered such controls in a single path. In this paper we extend this approach to more general aggregations of flows: for instance, emerging peer-to-peer (p2p) networks dynamically manage multiple connections with the common goal of a single download. Fairness in this context applies naturally to the overall download rate of flows within different routes. This leads us to formulate in Section II a NUM problem in terms of aggregate rates and we develop a variant of the primal-dual algorithm to obtain global convergence. This suggests a natural control law for the number of connections, which we develop in Section III.

Running this kind of control at the sources requires user self-restraint, which is against the incentive to open more connections; also, the number of connections can explode not only because of greedy users, but even in the case of a “neutral” stochastic load that happens to exceed network capacity [6]–[8]. In this situation, the network can protect its stability and strive for fairness through flow admission control, as we proposed originally in [4]. In Section IV we extend this work to the multiple path setting, and also consider the possibility of controlled routing at network admission. In Section V we develop packet-level implementations of some of these algorithms, demonstrated through ns2 simulations. Conclusions are given in Section VI.

II. USER-CENTRIC FAIRNESS

We consider a network composed of links, indexed by $l$, with capacity $c_l$, and a set of paths or routes, indexed by $r$. End-to-end connections (flows) travel through a single path. The resource allocation in this network is formulated in terms of users: user $i$ is a set $\{r: r \in i\}$ of connections that serve a common application. We establish the following notation:

- $x_r$ denotes the rate of a single connection on route $r$.
- $n_r$ denotes the number of connections on route $r$.
- $\varphi_r = n_rx_r$ is the aggregate rate on route $r$.
- $\varphi^i = \sum_{r \in i} \varphi_r$ is the aggregate rate of user $i$.

User $i$ values the aggregate rate $\varphi^i$ of its flows. This is expressed as in [1] by an increasing, concave utility function $U_i(\varphi^i)$. This aggregation can be very general, not restricted to multiple paths between a single source-destination pair.

Let $R$ be the standard routing matrix, with entries $R_{lr} = 1$ if route $r$ contains link $l$, and 0 otherwise. The aggregate rate in link $l$ from all users is $y_l = \sum_r R_{lr} \varphi_r$.

The desired user-centric fairness is given by the following social welfare maximization.

**Problem 1 (USER WELFARE):** Maximize $\sum_i U_i(\varphi^i)$, subject to link capacity constraints $y_l \leq c_l \forall l$.

Note that the above resource allocation depends not only on the rates of individual flows controlled by the transport protocol (TCP), but also on the number of active connections associated with each user. In this paper we take TCP as given, and focus on the control of the number of connections to achieve the desired fairness.

Before tackling the connection dynamics, assume first that users had direct control of the total rates per route $\{\varphi_r\}$.
Which decentralized dynamic control laws take the network to the desired allocation? For the case of a single path per user, the above problem is mathematically identical to standard congestion control theory, and has well known solutions. In the multipath situation, it is analogous to studies in multipath congestion control (e.g. [9], [10]), where difficulties arise due to the fact that the objective function is not strictly concave in the variables \( \phi_r \). Let us write the Lagrangian of the problem:

\[
L(\phi, p) = \sum_i U_i \left( \sum_{r \in i} \phi_r \right) - \sum_r q_r \phi_r + \sum_l p_l c_l,
\]

where \( p_l \) is the link price and \( q_r = \sum_{l \in r} p_l \) is the route price.

The Karush-Kuhn-Tucker conditions that characterize the saddle point of \( L \) imply that for each \( r \),

\[
U_i' \left( \phi_i^* \right) = q_i^* := \min_{r \in i} q_r^*;
\]

and user \( i \) only sends equilibrium traffic through minimum price paths. In general, the optimal rates need not be unique. It is also well known, going as far back as [11], that primal-dual dynamics, which are gradient laws to find a saddle point of the Lagrangian can induce oscillations (even if the equilibrium is unique), due to the lack of strict concavity.

A first contribution of this paper is to provide a stabilizing variation to multipath primal-dual dynamics, through an anticipative term in the prices. This “derivative action” has been used before in [12] for node-based multipath routing. The control law becomes:

\[
\dot{\phi}_r = \kappa_r(\phi_r) \left[ U_i'' \left( \sum_{r \in i} \phi_r \right) - q_r - \nu \dot{q}_r \right].
\]

\[
\dot{y}_i = \gamma_i(y_i - c_i) p_i^+,
\]

\[
y = R\phi, \quad q = R^T p.
\]

Here as usual \( \kappa_r(\phi_r) > 0 \), \( (\cdot)_+ \) is a projection that keeps the prices positive and \( \gamma_i \) is a positive constant. It is well-known (see e.g. [13]) that the optimum is globally asymptotically stable in the single-path case without the derivative action. Note that the equilibrium is unchanged by this addition.

We have the following result (c.f. [14] for the proof):

**Theorem 1:** Assume that there is a solution of Problem 1 where every link is saturated. Under the control law of equations (2) all trajectories converge to this solution.

**Remark 1:** The hypothesis of saturation in all links is restrictive. Note, however, that a local asymptotic stability result can be obtained in general, since unsaturated links can be removed from the local analysis (they generate no prices around equilibrium).

**III. CONTROLLING THE NUMBER OF CONNECTIONS**

In practical networks the aggregate rate of flows in a route is not directly controlled. Rather, a user may open a number of connections \( n_r \), each of which is assigned a rate \( x_r \), by TCP congestion control; the resulting aggregate \( \phi_r = n_r x_r \) follows

\[
\phi_r = n_r x_r
\]

indirectly from this procedure. The TCP layer thus provides a mapping \( \{n_r\} \rightarrow \{x_r\} \), which itself can also be modelled through convex optimization (see [13], Chapter 9):

**Problem 2 (TCP Congestion control):** For fixed \( \{n_r\} \),

\[
\max_{\{x_r\}} \sum_r n_r U_{TCP}(x_r),
\]

subject to capacity constraints \( \sum_l R_{tl} n_r x_r \leq c_l \) for each \( l \).

The above optimization amounts to “flow-level fairness”: each TCP flow is assigned a utility \( U_{TCP_r} \), and congestion control maximizes their sum. Note that \( U_{TCP_r} \) is a mathematical model for the underlying TCP protocol, and does not necessarily reflect user preferences, as does \( U_i \) in Problem 1.

Moreover, the induced mapping \( n_r \rightarrow \phi_r \) can be shown to be increasing in \( n_r \), a greedy user has incentives for opening multiple connections in order to enlarge its bandwidth share. This emphasizes the point that without consideration for the role of the \( n_r \) variables, there can be no meaningful notion of network fairness, nor control of its stability.

The problem we consider in this section is thus the following: suppose the TCP layer is given, modelled as in Problem 2. How should the number of connections per route \( n_r \) be controlled to achieve the user-centric fairness described previously? For the moment we will assume cooperation: users in the network are willing to engage in this control to achieve the desired fairness. In the following section we discuss alternatives for non-cooperative scenarios. We will use fluid models in which \( n_r \) are treated as continuous quantities; practical versions will require appropriate discretizations.

The problem under consideration is best explained through Figure 1: on the right, we represent the network by an entity that receives aggregate rates \( \phi_r \), and returns congestion prices \( q_r \) per route. These are used by the TCP module to generate the rate \( x_r \) per connection; thus the inner loop represents TCP congestion control, for fixed \( n_r \). What we wish to design here is the outer loop (which operates at a slower timescale), controlling the \( n_r \) such that the overall dynamics of \( \phi_r \) achieves the desired user-centric fairness.

In [4], [5] the authors considered the single path case, proposed certain control laws and obtained partial results. Here we develop alternative laws that allow for stronger results, as well as the extension to the multi-path setting.
We seek a control law for $n_r$ that results in the dynamics (2) for $\varphi_r$. Consider the following:

$$\dot{n}_r = n_r \left[ k(U'_r(\varphi^r) - q_r - \nu q_r) - \dot{x}_r/x_r \right].$$

(3)

With this choice for $\dot{n}_r$, we have:

$$\dot{\varphi}_r = n_r x_r + n_r \dot{x}_r = k \varphi_r \left( U'_r(\varphi^r) - q_r - \nu q_r \right)$$

which reduces to the dynamics (2) with $\kappa(\varphi_r) = k \varphi_r$.

For further clarity, and to facilitate implementation, it will be more convenient to rewrite the dynamics of $n_r$ in terms of the congestion price, eliminating the variable $x_r$ by invoking the KKT condition $U'_r(x_r) = q_r$ that corresponds to Problem 2. Focusing for instance on the $\alpha$-fair family of utilities [15], where $U'_r(x_r) = x_{\alpha}^{-\alpha}$ we have

$$\frac{\dot{x}_r}{x_r} = -\frac{1}{\alpha} \frac{q_r^{-1/\alpha - 1}}{q_r^{-1/\alpha} q_r} \dot{q}_r = -\frac{\dot{q}_r}{\alpha q_r}$$

and therefore the dynamics of $n_r$ becomes

$$\dot{n}_r = n_r \left[ k(U'_r(\varphi^r) - q_r - \nu q_r) + \frac{\dot{q}_r}{\alpha q_r} \right].$$

(4)

The following result is a direct corollary of Theorem 1.

**Proposition 2:** Assume the optimal allocation of Problem 1 saturates all links. Then the connection-level dynamics (4) globally stabilizes this equilibrium.

Observe that in equation (4), the predictive terms in $q_r$ play opposing roles. This suggests the simpler control law

$$\dot{n}_r = n_r \left[ k(U'_r(\varphi^r) - q_r) \right]:$$

(5)

in fact when translated to $\varphi_r$, this yields dynamics very similar to (2), with derivative action in the control of $\varphi_r$, except that the damping parameter $\nu_r = (\alpha k q_r)^{-1}$ is time varying and route dependent. Extending our earlier stability argument to this case remains open at this time. A similar law was studied in [4] showing only local stability properties.

We have thus a choice between a more complicated law (4), with guaranteed global properties, and the simpler (5), with local guarantees. In section Section V, we explore the properties of the latter by simulation.

IV. ADMISSION CONTROL AND ROUTING

The proposal in the previous section implies users are willing and able to control the number of connections for the sake of overall welfare. However, a greedy user may simply not cooperate. In this situation, the network should be able to respond by imposing admission control on the number of connections.

Previous work on admission control (e.g. [6]) has been motivated by the requirement of maintaining network stability under stochastic models of load. Suppose connections arrive at each route $r$ as a Poisson process with intensity $\lambda_r$, carrying an exponentially distributed workload of mean $1/\mu_r$. Jobs are served at a rate determined by TCP congestion control, modeled in terms of $\alpha$-fairness. As shown by [7], [8], the condition for stochastic stability for such a system is:

$$\sum_r R_r \rho_r < c_l \quad \forall l,$$

(6)

where $\rho_r = \lambda_r/\mu_r$ is the load on route $r$. This result is fairly independent of the TCP flavor. However, if condition (6) is not satisfied, the number of connections will grow without bounds, each one of them getting a tiny amount of bandwidth. This has led some authors to advocate admission control [6].

In our previous work [4], [5], we proposed an algorithm that combines admission control with fairness in a single path setting. In this section we extend these results to more general flow aggregates, we characterize the ensuing equilibria, and also propose new mechanisms where the admission decision is combined with a choice of route through the network.

A. Admission control

Based on the analysis of Section II, we develop a decentralized admission control rule that drives an overloaded system into a fair operating point, from a user-centric perspective. The admission control rule is simply:

- If $U'_i(\varphi^i) > q_r \rightarrow$ admit connection.
- If $U'_i(\varphi^i) \leq q_r \rightarrow$ discard connection.

(7)

At each connection arrival of user $i$ on route $r$, we decide whether to admit it based on the sign of $\dot{n}_r$ in (5).

Consider the stochastic traffic model described above, characterized by traffic loads per route. A fluid model for the resulting connection process is given by:

$$\dot{n}_r = \lambda_r \{ U'_i(\varphi^i) > q_r \} - \mu_r \varphi_r$$

(8)

Equation (8) can be interpreted as a large network asymptotic (fluid limit) of the corresponding Markov Process [5].

We now characterize the equilibrium points of these dynamics, combined with the map $n_r \rightarrow (\varphi_r, q_r)$ that results from the optimum of Problem 2.

**Proposition 3:** The equilibrium points are solutions of Problem 3 (Saturated USER WELFARE):

Maximize $\sum_i U_i(\varphi^i)$,

subject to $R \varphi \leq c$, and $\varphi_r \leq \rho_r$ for each $r$.

A proof, including a more formal discussion of the switching dynamics is deferred to [14]. Essentially, one must show that an equilibrium implies either

$$\varphi^*_r < \rho_r \quad \& \quad U'_i(\varphi^{*i}) = q_r^* \quad \text{or} \quad (9a)$$

$$\varphi^*_r = \rho_r \quad \& \quad U'_i(\varphi^{*i}) > q_r^* \quad \text{or} \quad (9b)$$

which are equivalent to the KKT conditions for Problem 3.

Note that when traffic demands $\rho_r$ are very large, Problem 3 reduces to Problem 1 and admission control is indeed imposing the desired user-centric notion of fairness. On the other hand, if some users have routes which demand less than their fair share according to Problem 1, the overall allocation protects them from routes in overload, which is a desirable property.


B. Combining admission control and routing

Assume now that the external demands are not associated with routes, but rather with users with an aggregate load \( \rho^i \), and it is the choice of the network or application to decide which route shall be used. Routing is now another control variable in the system: individual connections remain single-path, but the user/network has the choice to balance the number of active connections through multiple paths.

Let \( A_{ir} \) be the set of network states such that connections arriving from user \( i \) are routed through route \( r \in \{ r \} \). If no admission control is performed, a fluid model for the connection dynamics under the routing policy is

\[
\dot{n}_{ir} = \lambda_i 1_{A_{ir}} - \mu_i \varphi_r.
\]

At this point we have not yet chosen a policy. The only general requirement is that the sets \( A_{ir} \) are a partition of the space, i.e. \( \sum_{r \in \{ r \}} 1_{A_{ir}} = 1 \). The first question in this new model is to characterize when stochastic stability occurs, generalizing (6).

\[
\sum_r \alpha^i_r \leq c_l \quad \forall l.
\]

In other words, for stability there must exist a traffic split \( \rho_r = \alpha^i_r \rho \) of the user loads among routes that satisfies the non-strict version of (6).

\[
\text{Remark 2: Condition (11) was obtained in [16] for stochastic stability in the case of Multipath TCP. In that case it is the TCP layer that is modified to make simultaneous use of the available routes. Here, each connection remains single-path, and achieving stability is in the hands of the routing policy.}
\]

Note that given splits \( \alpha^i \) satisfying the strict version of (11), the random policy that chooses routes with these probabilities will be stabilizing. However, we wish to find a policy that does not depend on this prior knowledge. We propose the following: when a connection arrives for user \( i \),

\[
\text{route to } r^*_i = \arg \min_{r \in \{ r \}} q_r.
\]

This policy is natural: it tries to minimize the price seen by connections. Since congestion price is inversely related to connection rate, it is equivalent to choosing to route new flows to the route with best rate for individual connections. Formally, \( A_{ir} \) is the set of states \( n = \{ n_r \} \) such that the resulting TCP congestion prices satisfy \( q_i(n) = \min_{r \in \{ r \}} q_r(n) \), with a suitable tie breaking rule. We have the following partial result.

\[
\text{Proposition 5: For a network of parallel links (i.e. all routes have a single bottleneck), if (11) holds then policy (12) is stabilizing for the dynamics (10).}
\]

As before, in the case where the stability condition is not satisfied, in order to maintain stability and fairness, admission control is needed. The reasonable way to merge the results on admission control and routing is the following combined law:

Admit new connection if \( \min_{r \in \{ r \}} q_r < U_1(\varphi^i) \)

If admitted: route through cheapest path.

The combined routing-admission control dynamics is thus

\[
\dot{n}_{ir} = \lambda_i 1_{A_{ir}} 1_{\{U_1(\varphi^i) > \min_{r \in \{ r \}} q_r\}} - \mu_i \varphi_r,
\]

and we have the analog of Proposition 3:

\[
\text{Proposition 6: The equilibrium points of (13) are optimal solutions of:}
\]

\[
\text{Maximize } \sum_r U_i(q^i_r),
\]

subject to \( R\varphi \leq c \), and \( \varphi^i \leq \rho^i \) for each user \( i \).

The above NUM problem is similar to Problem 3 but with user demands \( \rho^i \) per user instead of demands per route.

V. PRACTICAL IMPLEMENTATION AND SIMULATIONS

In this section we discuss practical implementation issues and explore the performance of the policies developed through ns2 [17] simulations.

We first consider the topology of Figure 2, where we have two users that intend to download data from 3 servers. In order to introduce an imbalance between users and routes, some routes have a larger round trip time than others. We implemented a packet level version of the control law (5). Each user then begins with a single TCP/Newreno connection per route, and the congestion price \( q_r \) is taken as the packet loss probability along that route, measured by the users counting the fraction of retransmitted packets.

Fig. 2. Topology simulated in Scenario 1.

In this experiment each user periodically updates a variable with the target number of connections, by measuring the current values of \( \varphi^i \) and \( q_r \) and integrating (5). We chose \( U_1(\varphi) = U_2(\varphi) = \log(\varphi) \), which give, in equilibrium, \( \varphi^1 = \varphi^2 = 10 \text{Mbps} \). Each second, the user chooses whether to open or close a connection on route \( r \) by comparing \( n_r \) with the corresponding target. Results are shown in Figure 3. The number of connections on each route tracks an equilibrium value, and the total rate evolves reaching the fair allocation.

A second experiment consisted in applying the admission control law (7) instead of cooperative user control. Users generate connections as a Poisson process for each route, and loads are chosen such that the network is in overload. The results are shown in figure 4. As we can see, admission control also manages to drive the average user rates to the fair share of the USER WELFARE problem in this case.

We now consider a “parking lot” topology, with 2 links in tandem, of capacities 8 and 6 Mbps, and 3 routes, with single-path users: users 1 and 2 use respectively the single links 1 and 2, user 3 uses through both links.

Users generate connections according to a Poisson process, and the network performs admission control by the rule (7).
The results show that in overload, with all users having TCP/Newreno.

In the second situation, we assumed that the total load for $\alpha$-fair utilities with $\alpha = 5$ to approximate max-min fairness. Individual connections are TCP/Newreno.

The max-min allocation for this network is $\phi_1 = 5$Mbps and $\phi_2 = \phi_3 = 3$Mbps. In the first simulation the network is in overload, with all users having $\phi_i$ greater than their fair share. The results show $n_1 \approx 5$, $n_2 \approx 8$ and $n_3 \approx 15$ admitted connections, with total rates according to the first graph in Figure 5. The max-min allocation is approximately achieved.

In the second situation, we assumed that the total load for user 2 is below its fair share, at $\phi_2 = 1$Mbps, and is therefore saturated in the sense of Problem 3. The new average rates in this case are shown in the second graph of Figure 5. Here the admission control protects user 2 by allowing its share of 1 Mbps into the network, and reallocates the remaining capacity as in Problem 3 to the new equilibrium $\phi_1 = \phi_3 = 4$Mbps, $\phi_2 = 1$Mbps.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed a new paradigm for resource allocation in networks, which intends to bridge the gap between classical NUM applied to congestion control and the user-centric perspective for fairness. We showed that the number of connections can be used to achieve this fairness, either through cooperative control or through network admission control. Finally, we showed practical implementations of the proposed mechanisms.

Several theoretical questions are still open for future research, the most important being: global stability for multipath admission control, and the sufficiency of the stability condition for our routing policy. In terms of implementation, new congestion notification protocols will help make these decentralized admission control mechanisms scalable to large networks, and we plan to address this in the future.

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